Contents

1 Volume 1: Ethics and Quantitative Methods

1.1 Study Session 2: Quantitative Methods, Basic Concepts

1.1.1 Reading 5: The Time Value of Money

1.1.1.1 Interest Rates

LOS 5a.

Interpret interest rates as required rate of return, discount rate or oppurtunity cost.

The interest rate r is the required rate of return. We often call it the discount rate or the opportunity cost.

Why do we call it **opportunity cost**? If you could earn r percent when putting money on your bank account but you do not do it, then you do not earn r percent. Therefore, it is the cost of the alternative forgone (= opportunity).

LOS 5b.

Explain an interest rate as the sum of a real risk-free rate, expected inflation, and premiums that compensate investors for distinct types of risk.

We have

Interest Rate = Real-risk-free interest rate + Inflation premium + Default risk premium + Liquidity premium + Maturity premium

= Nominal risk-free interest rate + Default risk premium + Liquidity premium + Maturity premium

The **real risk-free interest rate** is a single-period interest rate, if the security is totally risk-free and no inflation is expected. **Inflation premium** compensates the investor for expected inflation. If inflation reduces the investor´s purchasing power, he wants to get a compensation, the inflation premium.

KEEP IN MIND:

Nominal risk-free rate = Real-risk-free interest rate + inflation premium.

The **default risk premium** is a compensation for the possibility that the borrower does not pay the contracted amount at the contracted time.

1.1.1.2 Future Value of Cash Flows

LOS 5e.

Calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an oridinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows.

We differentiate between

- future value of a single cash flow, and
- future value of series of cash flows.

1.1.1.2.1 Future value of a single cash flow

LAMBERT-RULE:

Question: What would you prefer? Getting an amount of € 100 today or € 105 in one year?

Answer: it depends on the interest rate!

If your bank offers you an interest rate of r = 6 percent, you should ask for € 100 today, put them on your bank account and have € 106 in one year.

If your bank offers you an interest rate of only r 4 percent, then you should ask for € 105 in one year, because € 100 today, put on your bank account, would pay only € 104 in one year.

KEEP IN MIND:

Therefore, the task of an interest rate is to make current and future currency amounts equivalent based on their time value.

If an amount of money, say PV, is compounded at an interest rate r, we get an amount of FV_1 after one period, namely

$FV_1 = PV \cdot (1 + r)$, and

$FV_N = PV \cdot (1 + r)^N$ after N periods.

So the future value equals the present value times the future value factor $(1 + r)^N$.

Example 1:

Tom B., living in Paris, France, puts € 1,000 on his bank account. He gets 3 percent per year, compounded annually. How much money can he expect

a) in one year,

b) four years from now?

a) In one year, he will earn

$FV_1 = PV \cdot (1 + r) = 1{,}000 \cdot (1 + 0.03) = 1{,}000 \cdot 1.03 =$ € 1,030.

b) One year later, he will have $FV_2 = PV \cdot (1 + r) = 1{,}030 \cdot (1 + 0.03) =$ € 1,060.90. At the end of year 3, he will have $1{,}060.9 \cdot 1.03 =$ € 1,092.73, and one year later, it is going to be € 1,125.51. The latter can also bge computed directly via

$$\begin{aligned} FV_N &= PV \cdot (1 + r)^N \\ &= 1{,}000 \cdot (1 + 0.03)^4 \\ &= 1{,}000 \cdot 1.125508 \\ &= \text{€ } 1{,}125.51. \end{aligned}$$

LOS 5c.

Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding.

If we compound with more than one compounding period per year, we get

$FV_N = PV \cdot (1 + r_S/m)^{m \cdot N}$.

This is the future value after N years, if the amount is compounded m times a year with an annual interest rate of r_S.

Example 2:

Tom´s amount of money of example 1 is compounded quarterly instead of annually. How much money does he have after four years?

Using N = 4, r = 0.03, and m = 4, we get

$$\begin{aligned} FV_N &= PV \cdot (1 + r_S/m)^{m \cdot N} \\ &= 1{,}000 \cdot (1 + 0.03/4)^{4 \cdot 4} \\ &= 1{,}000 \cdot 1.126992 \\ &= € \ 1{,}126.99. \end{aligned}$$

KEEP IN MIND:

The amount of example 2 (€ 1,126.99) is superior to the one of example 1 (€ 1,125.51), because we compound more often.

Example 3:

Tom´s amount of money of example 1 is compounded monthly and daily. Compute the future value after four years.

a) Using m = 12, we get

$$\begin{aligned} FV_4 &= PV \cdot (1 + r_S/m)^{m \cdot N} \\ &= 1{,}000 \cdot (1 + 0.03/12)^{12 \cdot 4} \\ &= 1{,}000 \cdot 1.127328 \\ &= € \ 1{,}127.33. \end{aligned}$$

b) Now we have m = 360, we calculate

$$\begin{aligned} FV_4 &= PV \cdot (1 + r_S/m)^{m \cdot N} \\ &= 1{,}000 \cdot (1 + 0.03/360)^{360 \cdot 4} \\ &= 1{,}000 \cdot 1.127491 \\ &= € \ 1{,}127.49. \end{aligned}$$

KEEP IN MIND:

The future value does not rise infinitely, if we raise the number m from 1 to 4, 12, 360, and so on. If will not raise above the future value that arises from continuous compounding.

If **compounded continuously**, we get

$FV_N = PV \cdot e^{rS \cdot N}$.

Example 4:

Tom´s amount of example 1 is compounded continuously. Compute the future value in year 4.

We compute

$$\begin{aligned} FV_4 &= PV \cdot e^{rS \cdot N} \\ &= 1{,}000 \cdot e^{0.03 \cdot 4} \\ &= 1{,}000 \cdot 1.1274969 \\ &= 1{,}127.50. \end{aligned}$$

LOS 5c.

Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding.

We have to deal with stated and effective rates now in order to fully understand LOS 5c. If the annual interest rate r is stated, we do not directly see the interest rate that matters in case of compounding m times during the year. Therefore, we have to calculate

- the periodic interest rate, and, after that,
- the effective annual rate EAR
  - ◊ with discrete compounding and
  - ◊ with continuous compounding.

The **periodic interest rate** is computed as r/m, the **effective annual rate** as

$EAR = (1 + r/m)^m - 1$, in the case of discrete compounding, and

$EAR = e^{rS} - 1$ in the case of continuous compounding.